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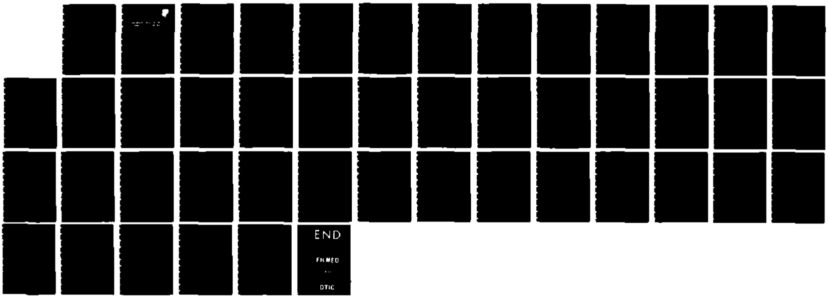

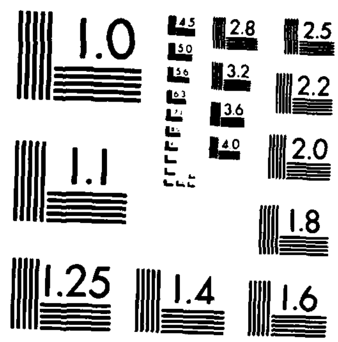
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NON-UNIFORM CURRENTS ON A WEDGE ILLUMINATED BY A TE PLANE WAVE

University of Dayton

P. K. Murthy and Gary A. Thiele

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Closed form expressions for non-uniform currents on a perfectly conducting, infinite wedge illuminated by transverse electric (TE) plane wave are presented. These expressions are derived by requiring that they coincide with the current predicted by the asymptotic diffraction theory far from the edge and, further, that they agree with the current predicted by the eigenfunction solution at the edge. The angle of incidence is arbitrary and our expressions remain valid even for glancing angles of incidence when either one or both faces of the wedge is in the vicinity of a Geometric optic boundary.

Formulas presented here are simple involving the well known modified Fresnel functions but are not uniform. Exact expressions for non-uniform currents are available for the two special cases of half plane and infinite plane. For these special cases, our solution reduces to the exact solution. Currents computed using the expressions developed here are compared with currents computed from the eigenfunction solution of the wedge. Good agreement is obtained throughout.

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I. INTRODUCTION

Two of the principal theories of edge diffraction are the geometrical theory of diffraction (GTD) enunciated by Keller [1] - [3] and the physical theory of diffraction (PTD) originated by Ufimtsev [4]. The GTD extends the Geometrical Optics (GO) by the inclusion of rays diffracted by surface singularities. Keller's theory fails in the vicinities of GO boundaries known as transition regions. To overcome this difficulty, the uniform theory of diffraction (UTD) [5], [6] and the uniform asymptotic theory (UAT) [7] - [9] have been devised. A common failing of all ray optic techniques is that they predict infinite fields at caustics. The method of equivalent currents (MEC) alleviates the problem at caustics encountered by GTD and a number of authors have made contributions towards this end [10] - [14]. This method is based upon prescribing fictitious currents on the true surfaces.

In contrast to GTD, PTD is a technique based upon integrating the currents induced on the scatterer. Ufimtsev postulates that the induced current is a sum of uniform or physical optics (PO) current induced by the GO surface field and the non-uniform current induced by the diffracted field at the surface. The scattered field is obtained as a surface integral of these currents. Thus, PTD is an extension of PO. It must be noted, however, Ufimtsev does not give explicit expressions for the non-uniform currents, but instead determines the field due to these currents from "indirect considerations".

Knott and Senior [15] present an elegant summation of the three techniques, GTD, PTD and MEC. Lee [16] compared the UAT with PTD and points out that the lack of explicit expressions for non-uniform currents or its dominant contribution is a notable disadvantage of Ufimtsev's theory. Schretter and Bolle [17] have attempted to find closed form approximations for surface currents on a wedge for both TE and TM polarizations. Their expressions suffer from significant discrepancies, however. A notable addition to the literature on edge diffraction is a recent review paper by Deschamps [18].

In this paper, we present closed form expressions for non-uniform currents on a wedge illuminated by a TE-plane wave. These expressions are derived by requiring that they coincide with the current predicted by the asymptotic diffraction theory far from the edge and, further, that they agree with the current predicted by the eigenfunction solution at the edge. The angle of incidence is arbitrary and our expressions remain valid even in those cases where either one or both faces of the wedge is in the vicinity of a GO boundary. Exact expressions for non-uniform currents are available for the two special cases of a half plane and an infinite plane. For these special cases, our expressions reduce to the exact solution. Extensive computations have been made and our expressions are compared with non-uniform currents computed from an eigenfunction solution of the wedge. Good agreement is obtained throughout.

2. THEORY

In this section, we obtain expressions for surface currents induced on an infinite wedge by a normally incident Transverse Electric (TE) plane wave. Figure 1 shows the geometry of the problem. A cylindrical coordinate system is used with the edge of wedge coinciding with the z-axis. Face A is defined by the $\phi = 0$ plane and Face B is defined by the $\phi = (2\pi-\alpha)$ plane, where α is the inferior wedge angle. It is convenient to define the numbers 'n' and 'ν' such that

$$(2\pi-\alpha) = n\pi = \frac{\pi}{\nu}, \; 1 < n < 2 \text{ and } \frac{1}{2} < \nu < 1 \tag{1}$$

$n = 1$ ($\nu = 1$) corresponds to an infinite plane and $n = 2$ ($\nu = \frac{1}{2}$) corresponds to a half plane.

The magnetic field of the incident TE-plane wave is given by,

$$\bar{H}_i = H_o \, e^{-j\beta\rho \cos(\phi - \phi_i)} \, \hat{u}_z \tag{2}$$

H_o is the amplitude of the plane wave, β is the propagation constant, ϕ_i is the angle of incidence as shown in Figure 1 and $\hat{u}_z$ is the unit vector in the z-direction. The unit tangential vectors $\hat{t}_A$ and $\hat{t}_B$ and the unit normal vectors $\hat{n}_A$ and $\hat{n}_B$ are shown in Figure 1. Also shown in Figure 1 are the reflection boundary and the shadow boundary and the transition regions associated with these boundaries.

From the symmetry of the problem under consideration we need to consider the angles of incidence in the range $0 < \phi_i < n\frac{\pi}{2}$ only.

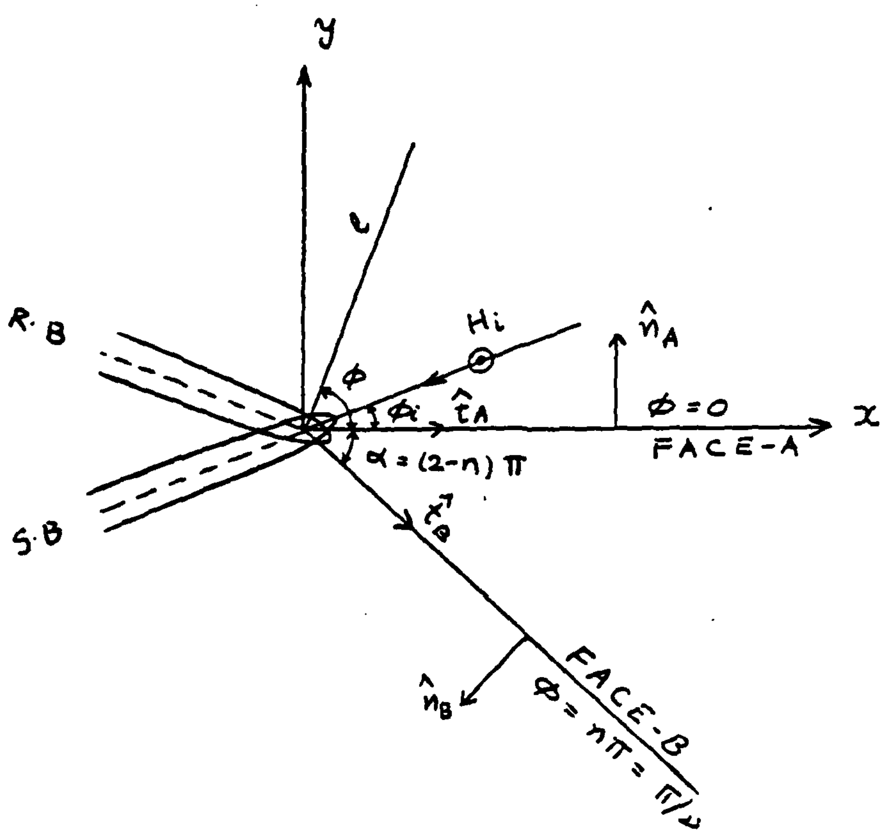

Figure 1. Geometry of a Wedge.

2.1. Formulation

The current induced on either face of tne wedge is considered to consist of two components. Following the terminology of Ufimtsev [4], these components of the current are labeled the unifrom (physical optics) and non-uniform currents (NUC). Thus,

$$\vec{J} = \vec{J}_u + \vec{J}_{nu} \tag{3}$$

The total field of a wedge may be decomposed into the Geometrical Optics (GO) field and the diffracted field. The uniform and non-uniform currents may be considered to be induced by the GO field and the diffracted field. Hence, the uniform current is,

$$\begin{aligned} \vec{J}_u &= 2\hat{n} \times \vec{H}_i \quad && \text{in the illuminated region} \\ &= 0 && \text{in the shadow region} \end{aligned} \tag{4}$$

The non-uniform current is not so easily obtained. However, far from the edge, ($\beta\rho >> 1$), the diffracted magnetic field and hence the current may be ootained using the asymptotic theory of diffraction [5], [6]. Hence,

$$\vec{J}_{nu} = \hat{n} \times \vec{H}_d \qquad \beta\rho >> 1 \tag{5}$$

At the edge ($\rho = 0$), the eigenfunction solution may be used to obtain the total magnetic field. From James [19], the total field is,

$$\vec{H}_{Total}(\rho = 0) = 2\nu H_0 \hat{u}_z \tag{6}$$

The non-uniform current is simply the difference between the total and uniform currents and since the latter is known,

$$\vec{J}_{nu}(\rho = 0) = 2\nu H_o(\hat{n} \times \hat{u}_z) - \vec{J}_u(\rho = 0) \tag{7}$$

Thus, from Equations (5) and (7), the non-uniform current is obtained for points well removed from the edge and at the edge. We now postulate that the NUC may be expresed in the following fashion:

$$\vec{J}_{nu} = h(\beta\rho;\phi_i,n)\ e^{-j\beta\rho}\ \hat{t} \tag{8}$$

where 'h' is a function yet to be determined and $\hat{t}$ is an appropriate unit tangent vector. We require that for $\beta\rho >> 1$, it must reduce to Equation (5) and further, for $\beta\rho = 0$, it must reduce to Equation (7). Thus, the problem consists in finding the function $h(\beta\rho;\phi_i,n)$ in such a way that for $\beta\rho >> 1$, Equation (8) gives the same current as that predicted by the asymptotic theory of diffraction and for $\beta\rho = 0$, it yields the same current as that dictated by the eingenfunction solution.

Out of a wide class of possible functions, the choice for $h(\beta\rho)$ may be narrowed down to one class of functions by an examination of the physics of the problem. As shown in Figure 1 and as pointed out by James [19], the edge and its vicinity is always in the transition region associated with GO boundaries. Thus, the function $h(\beta\rho)$ acts as a connecting function between points that are in and out of a transition region. In such a situation, a Fresnel function or its variations represented the field variation as demonstrated by both UTD [5] and UAT [7]. The Fresnel function appears in

Sommerfeld's solution for a half plane [20] and also Pauli's solution for the wedge [21]. The present case is no exception.

The particular function we use is the modified Fresnel function (MFF) denoted by $K_{-}(x)$. As in [19] it is defined by,

$$K_{-}(x) = \sqrt{\frac{j}{\pi}}\, e^{jx^2} \int_{x}^{\infty} e^{-jt^2}\, dt \tag{9}$$

The large and small argument approximations for MFF are:

$$K_{-}(x) \sim \frac{1}{2\sqrt{\pi j} \cdot x} \qquad x \rightarrow \infty \tag{10}$$

$$K_{-}(x) \sim \frac{1}{2} - \sqrt{\frac{j}{\pi}} \cdot x \qquad x \rightarrow 0 \tag{11}$$

and

$$K_{-}(o) = \frac{1}{2} \tag{12}$$

Note that the phase changae in MFF as x changes from zero to infinity is $-\frac{\pi}{4}$ radians. As will be shown later, the phase change of NUC other than that due to $\exp(-j\beta\rho)$ is also the same.

Of special interest, for the purposes of this paper, is the MFF in the form $A\,K_{-}(Ax)$ where A is a real, positive constant. The small and large argument approximations for this function are,

$$A\,K_{-}(Ax) = \frac{A}{2} \qquad Ax = 0 \tag{13}$$

$$A\,K_{-}(Ax) \sim \frac{1}{2\sqrt{\pi j} \cdot x} \qquad Ax >> 1 \tag{14}$$

Note that the asymptotic approximation is the same for $A\,K_-(Ax)$ and $K_-(x)$. We will now derive the closed form expressions for non-uniform currents.

2.2 Only Face-A is Illuminated

When the angle of incidence is in the range $0 < \phi_i < (n-1)\pi$ only Face A is illuminated. Hence, the total current on Face A consists of both the uniform and non-uniform currents. Face B, being in the shadow region, has no uniform current and the total current consists of the non-uniform current only.

<u>Current on Face B</u>: The diffracted magnetic field at the edge, from the eigenfunction solution is given by

$$H^B_{z,d} = 2\nu\, H_o \qquad \rho_B = 0 \tag{15}$$

ρ_B is the distance from the edge along Face B. The diffracted magnetic field, far from the edge, using Keller's Theory of diffraction as in [5] is given by

$$H^B_{z,d} \sim 2\nu\, H_o \frac{\sin \nu\pi}{\cos\nu\pi + \cos\nu\phi_i} \frac{1}{\sqrt{2\pi j\beta\rho_B}} \qquad \beta\rho_B >> 1 \tag{16}$$

$$H^B_{z,d} = 4\nu\, H_o \frac{B}{2}, \qquad \beta\rho_B = 0 \tag{17}$$

$$\sim 4\nu\, H_o \frac{1}{2\sqrt{\pi j}} \frac{1}{x_B} e^{-j\beta\rho_B} \qquad \beta\rho_B >> 1 \tag{18}$$

where

$$B = 1$$

$$x_B = \sqrt{2\beta\rho_B}\, \frac{1}{F_B} \tag{19}$$

$$F_B = \sin\nu\pi/(\cos\nu\pi + \cos\nu\phi_i)$$

Comparing Equations (17) and (18) with Equations (13) and (14) and noting that F_B and hence x_B remain positive for $0 < \varphi_i < (n-1)\pi$, $H_{z,d}^{B}$ may be expressed in terms of MFF as,

$$H_{z,d}^{B} = 4\nu\ H_o\ B\ K_-(Bx_B)\ e^{-j\beta\rho_B} \qquad 0 < \rho_B < \infty \tag{20}$$

The non-uniform current is given by,

$$\vec{J}_{nu}^{B} = -\ 4\nu\ H_o\ B\ K_-(Bx_B)^{-j\beta\rho_B}\ \hat{t}_B, \quad 0 < \rho_B < \infty \tag{21}$$
$$0 < \phi_i < (n-1)\pi$$

By using Equations (10) and (12), we can easily verify that Equations (20) and (21) agree with the asymptotic theory of diffraction for $\beta\rho_B >> 1$ and with the eigenfunction solution at the edge. We expect that they constitute a close approximation to the true values for intermediate values of $\beta\rho_B$ and will so demonstrate by numerical computation in Section 3.

<u>Current on Face A</u>: The diffracted magnetic field at the edge and far from the edge is given by,

$$H_{z,d}^{A} = -\ 2\ H_o(1 - \nu)\ , \qquad \beta\rho_A = 0 \tag{22}$$

$$\sim\ 2\ H_o\ \nu\ F_A \frac{1}{\sqrt{2\pi j\beta\rho_A}}\ e^{-j\beta\rho_A}, \quad \beta\rho_A >> 1 \tag{23}$$

where

$$F_A = \frac{\sin n\nu\pi}{\cos\nu\pi - \cos\nu\varphi_i} \tag{24}$$

Noting that F_A remains negative for $0 < \phi_i < \pi$, Equations (22) and (23) may be cast in the form,

$$H_{z,d}^{A} = -2\, H_o\, (A/2) \qquad \beta\rho_A = 0 \tag{25}$$

$$\sim -2\, H_o \frac{A}{2\sqrt{\pi j}} \frac{1}{A\, x_A} e^{-j\beta\rho_A} \qquad \beta\rho_A >> 1 \tag{26}$$

where,

$$A = 2(1 - \nu)$$

$$x_A = \sqrt{\beta\rho_A/2}\; \frac{1}{\nu|F_A|} \tag{27}$$

As before, comparing Equations (25) and (26) to Equations (13) and (14), the diffracted magnetic field and the non-uniform current may be expresed in closed form as,

$$H_{z,d}^{A} = -2\, H_o\, A\, K_{-}(A\, x_A)\, e^{-j\beta\rho_A},\ 0 < \rho_A < \infty\ ;\ 0 < \phi_i < \pi \tag{28}$$

and $$\vec{J}_{nu}^{A} = -2\, H_o\, A\, K_{-}(A\, x_A)\, e^{-j\beta\rho_A}\, \hat{t}_A,\ 0 < \rho_A < \infty;\ 0 < \phi_i < \pi \tag{29}$$

Again, we can readily verify that our closed form expressions agree with the eigenfunction solution and the asymptotic theory of diffraction at the edge and far from the edge.

Since Keller's theory has been used to obtain Equations (21) and (29), they are valid only when neither Face A nor Face B is close to Geometrical optic boundaries. For instance, when $\phi_i \sim (n-1)\pi$, Face B is in the vicinity of a shadow boundary and Equation 16 is not applicable, and neither is Equation (21). Such cases, where either or both of the wedge faces are in the proximity of a GO boundary are dealt with in Section 2.4. Next, we consider the case when both faces are illuminated.

2.3 Both Faces Illuminated

We now consider the angles of incidence in the range $(n-1)\pi < \phi_i < \frac{n\pi}{2}$. In this range, both faces are illuminated. The current on Face A is still given by Equation (29). However, the current on Face B now consists of both uniform and non-uniform components.

Current on Face B: As before, the diffracted magnetic field at and far from the edge is given by,

$$H_{z,d}^{B} = -2 H_o (1 - \nu) \qquad \beta\rho_B = 0 \tag{30}$$

$$\sim 2 H_o \nu F_B \frac{1}{\sqrt{2\pi j \beta\rho_B}} e^{-j\beta\rho_B} \qquad \beta\rho_B >> 1 \tag{31}$$

where,

$$F_B = \frac{\sin\nu\pi}{\cos\nu\pi + \cos\nu\phi_i} \tag{32}$$

Note that F_B is negative for $(n-1)\pi < \phi_i < n\pi$. Clearly, Equations (30) to (32) are similar to Equations (22) to (24) and the closed form expressions for the field and the current are,

$$H_{z,d}^{B} = -2 H_o B K_-(B x_B) e^{-j\beta\rho_B}, \; 0 < \rho_B < \infty; \; (n-1)\pi < \phi_i < \frac{n\pi}{2} \tag{33}$$

and $$\vec{J}_{nu}^{B} = 2 H_o B K_- (B x_B) e^{-j\beta\rho_B} \hat{t}_B, \; 0 < \rho_B < \infty; \; (n-1)\pi < \phi_i < \frac{n\pi}{2} \tag{34}$$

where,

$$B = 2\,(1 - \nu) \tag{35}$$

$$x_B = \sqrt{\beta\rho_B/2}\quad (1/\nu\ |F_B|)$$

Now, it only remains to consider the cases where either or both of the faces are in a transition region associated with GO boundaries.

2.4 Transition Region Currents

Face B of the wedge is in a transition region in the following two cases:

a) when ϕ_i is close to but less than $(n-1)\pi$, a shadow boundary is close to Face B, and

b) when ϕ_i is close to but greater than $(n-1)\pi$, a reflection boundary is close to Face B.

In a similar fashion, Face A of the wedge is close to a GO boundary when ϕ_i is close to π. Both faces of the wedge may be close to GO boundaries; this condition occurs when n is close to two. Then $(n-1)\pi \sim \pi$. Hence, $\phi_i \sim (n-1)\pi$ imples $\phi_i \sim \pi$.

All these cases are considered in this section. The procedure used is essentialy the same as before. The only difference is that the diffracted field far from the edge is obtained using Pathak and Kouyonmjian's uniform theory of diffraction [5].

<u>Current on Face B</u>: The angles of incidence of interest are in the vicinity of $(n-1)\pi$. Let,

$$\phi_i = (n-1)\pi \pm \delta \tag{36}$$

where δ is a small positive quantity. The positive sign in Equation (36) corresponds to the case of Face B being illuminated with a reflection boundary being close by. The negative sign corresponds to the case when Face B is in the shadow region with a shadow boundary being close by. From UTD, the diffracted magnetic field far from the edge is given by,

$$H^B_{z,d} \sim \frac{H_o}{n\sqrt{2\pi j\beta\rho_B}} [\, \mathrm{Tan}\, \psi^- F(x^+) + \mathrm{Tan}\, \psi^+ F(x^-)]\, e^{-j\beta\rho_B} \qquad \beta\rho_B >> 1 \tag{37}$$

where, the transition function F(x) is given by [5],

$$F(x) = 2j\, |\sqrt{x}|\, e^{jx} \int_{|\sqrt{x}|}^{\infty} e^{-jt^2}\, dt$$

$$= 2\sqrt{\pi j x}\, K_-(|\sqrt{x}|) \tag{38}$$

and

$$\psi^{\pm} = \frac{\pi \pm \phi_i}{2n}$$

$$x^{\pm} = 2\beta\rho_B \cos^2\left(\frac{n\pi \pm \phi_i}{2}\right) \tag{39}$$

The small and large argument approximations of the transition function are given by,

$$F(x) \sim \sqrt{\pi j x} - 2\, jx \qquad x \to 0$$

$$\text{and} \quad F(x) \sim 1 \qquad x \to \infty \tag{40}$$

The second term in the square barckets of Equation (37) is noteworthy. For small 'δ',

$$\mathrm{Tan}\ \psi^{+} = \mathrm{Tan}\ \left(\frac{n\pi \pm \delta}{2n}\right) \sim \mp \frac{2n}{\delta} \tag{41}$$

$$\text{and} \quad x^{-} \sim \beta\rho_B \frac{\delta^2}{2} \tag{42}$$

Thus x^{-} is small and using the small argument approximation of $F(x^{-})$,

$$\mathrm{Tan}\psi^{+}\ F(x^{-}) \sim \mp \left[\ n \sqrt{2\pi j\beta\rho_B} - j\ 2n\ \beta\rho_B\delta\ \right] \tag{43}$$

Unlike x^{-}, for $\beta\rho_B$ large, x^{+} remains large and $F(x^{+})$ may be approximated by unity and the diffracted field may be written as,

$$H_{z,d}^{B} \sim u_1 + u_2 \qquad \beta\rho_B >> 1 \tag{44}$$

where,

$$u_1 = H_o \frac{\tan\psi^{-}}{n} \frac{1}{\sqrt{2\pi j\beta\rho_B}} e^{-j\beta\rho_B} \qquad \beta\rho_B >> 1 \tag{45}$$

$$\text{and} \quad u_2 = \mathrm{sgn}\ [\tan\psi^{+}]\ 2\ H_o\ \left\{ \frac{1}{2} - \sqrt{\frac{j}{\pi}} \sqrt{\frac{\beta\rho_B}{2}}\ \delta \right\}\ e^{-j\beta\rho_B},\ \beta\rho_B >> 1 \tag{46}$$

Keeping Equation (41) in mind, Equation (46) may be rewritten as,

$$u_2 = \mathrm{sgn}\ [\cot\psi^{+}]\ 2\ H_o\ \left\{ \frac{1}{2} - \sqrt{\frac{j}{\pi}} \sqrt{\frac{\beta\rho_B}{2}}\ 2n\ |\ \cot\psi^{+} |\ \right\}\ e^{-j\beta\rho_B},\ \beta\rho_B >> 1 \tag{47}$$

At the edge, as before,

$$H_{z,d}^{B} = 2\nu H_o - 2 H_o \text{ at } \rho = 0 \text{ if Face B is illuminated.} \quad (48)$$

$$= 2\nu H_o \qquad \text{at } \rho = 0 \text{ if Face B is shadowed.} \quad (49)$$

Comparing the expression in curly brackets of Equation (47) with Equation (11), it is easily recognized to be $2 H_o \operatorname{sgn}(\cot\psi^{+})$ $K_{-}[\sqrt{\beta\rho_B/2}\, 2n \,|\cot\psi^{+}|\,]$ for $\beta\rho_B >> 1$. For $\rho_B = 0$, i.e. at the edge, this function has a value of $\operatorname{sgn}(\cot\psi^{+}) H_o$. Keeping this in mind, the diffracted field at the edge may be rewritten as,

$$H_{z,d}^{B} = u_1 + u_2 \qquad \text{at } \rho = 0 \quad (50)$$

where,

$$u_1 = (2\nu - 1) H_o \qquad \text{at } \rho = 0 \quad (51)$$

and $u_2 = \operatorname{sgn}(\cot\psi^{+}) H_o \qquad$ at $\rho = 0 \qquad$ (52)

Now, Equations (51) and (45) may be combined together as in the preceding sections to yield,

$$u_1 = 2 H_o B K_{-}[B x_B^{-}] e^{-j\beta\rho_B} \qquad 0 < \rho_B < \infty \quad (53)$$

where,

$$B = (2\nu - 1)$$

$$\text{and } x_B^{-} = \sqrt{\frac{\beta\rho_B}{2}}\, 2n \cot\psi^{-} \quad (54)$$

Equations (52) and (47) yield, as already determined,

$$u_2 = 2\, H_o\, \mathrm{sgn}(\cot\, \psi^+)\; K_-(x_B^+)\; e^{-j\beta\rho_B} \tag{55}$$

where

$$x_B^+ = \sqrt{\frac{\beta\rho_B}{2}}\; 2n\; |\; \cot\, \psi^+\; | \tag{56}$$

The current on Face B is now obtained to be,

$$\vec{J}_{nu}^{\,B} = -\, 2\, H_o\, [B\; K_-(Bx_B^-) + \mathrm{sgn}(\cot\psi^+)\; K_-(x_B^+)]\; e^{-j\beta\rho_B}\; \hat{t}_A \tag{57}$$

$$0 < \rho_B < \infty$$

The analysis for current on Face A with $\phi_i \sim \pi$ remains essentially the same. We give, without details,

$$\vec{J}_{nu}^{\,A} = 2\, H_o\, [\; A\; K_-(Ay_A^+) - \mathrm{sgn}(\tan\psi^-)\; J_-(y_A^-)\;]\; e^{-j\beta\rho_A}\; \hat{t}_A \tag{58}$$

$$0 < \rho_A < \infty$$

where,

$$A = (2\nu - 1)$$

$$y_A^{\pm} = \sqrt{\frac{\beta\rho_A}{2}}\; 2n\; |\; \tan\psi^{\pm}\; | \tag{59}$$

We will now obtain a quantitative criterion as to when 'δ' may be considered small and, hence, use the expressions for transition region currents. Equation (41) is obtained by replacing the tangent function by its small argument approximation. Such an approximation may be taken to be valid for angles less than $\pi/20$. Hence,

$$\frac{\delta}{2n} < \frac{\pi}{20}$$

$$\text{and } \delta < \frac{n\pi}{10} \tag{60}$$

Note that the "width" of the transition region depends upon the wedge angle. Equation (57) is valid when $(n-1)\pi - \frac{n\pi}{10} < \phi_i < (n-1)\pi + \frac{n\pi}{10}$ and Equation (58) is valid when $\pi - \frac{n\pi}{10} < \phi_i < \pi + \frac{n\pi}{10}$. For all other angles of incidence, expressions derived in Sections 2.2 and 2.3 apply.

2.5 SPECIAL CASES

In the case of an infinite plane ($n = 1$) and a half plane ($n = 2$), exact solutions for induced currents are available in the literature [19]. We now show the expressions derived in this paper reduce to these exact solutions.

a. Infinite Plane: In the case of an infinite plane, both faces are always illuminated and Equations (29) and (34) are applicable. Clearly, both these non-uniform currents reduce to zero as they should. The total current on the infinite plane is simply the uniform current.

A more interesting case is when ϕ_i is zero and n is close to but slightly greater than unity. Then, Face B is in the shadow region and is close to a shadow boundary. Hence, Equation (57) is applicable. As n approaches unity, the wedge approaches an infinite plane and the shadow-side current must approach the uniform current. From Equations (54) and (56), $B \to 1$, $x_B^{\pm} \to 0$ and $\hat{t}_B \to -\hat{t}_A$ as $n \to 1$. Therefore, Equation (57) simplifies to

$$\vec{J}_{nu}^{\,B} = 2\, H_o\, e^{-j\beta\rho_B}\, \hat{t}_A \qquad \text{as } n \to 1 \tag{61}$$

This is simply the PO current and we obtain the exact solution. The non-uniform current on Face A, given by Equation (29) becomes zero, and the total current is simply the uniform current.

b. Half-Plane: The total non-uniform current on Face A is a sum of the non-uniform currents on Face A and Face B given by Equations (21) and (29). Noting that, for this case, $\hat{t}_B = \hat{t}_A$,

$$\vec{J}_{nu} = -\,2\, H_o\, [\, 2\nu\, B\, K_-(Bx_B) + A\, K_-(Ax_A)\,]\, e^{-j\beta\rho_A}\, \hat{t}_A,\ n = 2 \tag{62}$$

For $n = 2$, Equations (19) and (27) reduce to,

$$B = 1 = A$$

$$x_A = \sqrt{2\beta\rho}\, \left| \cos \frac{\phi_i}{2} \right| = x_B$$

and $\rho_A = \rho_B = \rho$.

Using the definition of the modified Fresnel function (Equation 9),

$$\vec{J}_{nu} = -\,4\, H_o \sqrt{\frac{j}{\pi}}\, e^{j\beta\rho\,(2\cos^2\frac{\phi_i}{2} - 1)} \int_{\sqrt{2\beta\rho}\,\left|\cos\frac{\phi_i}{2}\right|}^{\infty} e^{-jt^2}\, dt\ \hat{t}_A \tag{63}$$

Noting that the uniform current on the half plane is given by,

$$\vec{J}_u = 2\, H_o\, e^{j\beta\rho\, \cos\phi_i}\, \hat{t}_A$$

Equation (63) becomes,

$$\vec{J}_{nu} = -\, 2\, \vec{J}_u \sqrt{\frac{j}{\pi}} \int_{\sqrt{2\beta\rho}\, \left| \cos \frac{\phi_i}{2} \right|}^{\infty} e^{-jt^2}\, dt \tag{64}$$

This is exactly the expression obtained by James [19] for what he calls the correction current.

3. NUMERICAL ILLUSTRATIONS

The currents induced on a wedge are computed using the expressions derived in this paper. These results are compared to the exact currents obtained using the eigenfunction solution to test their accuracy.

We consider three separate wedge angles corresponding to $\alpha = \pi/12$, $\pi/2$, and $2\pi/3$. The case $\alpha = \pi/12$ is chosen because for this case both the faces of the wedge could be in transition regions associated with GO boundaries. The cases $\alpha = \pi/2$ and $2\pi/3$ are chosen because Shretter and Bolle's [17] curves correspond to these wedge angles. For each wedge angle, computations have been carried out for a range of angles of incidence.

We follow a uniform notation for presenting our results. The exact currents obtained from the eigenfunction solution are denoted by a dashed line (Face A) or solid line (Face B). Currents computed using the expressions derived in this paper are denoted by symbols only, triangles (△ △ △ △) for transition region currents and crosses (x x x) otherwise.

<u>Illustration 1:</u> $\alpha = 15^0$

We consider four angles of incidence, $\phi_i = 0°$, 135°, 166° and 172.5°. The first two angles of incidence correspond to only Face A being illuminated and the last two correspond to the case where both faces are illuminated.

Figure 2 shows the currents for the case of $\phi_i = 0^0$. Note that neither of the faces is in the vicinity of a GO boundary. Figure 3 corresponds to $\phi_i = 135^0$. Note that a shadow boundary is now in the vicinity of Face B, but Face A is not affected by any GO boundary. Figure 4 shows the currents for $\phi_i = 166^0$. For this case, both the faces are affected by a reflection boundary. Figure 5 corresponds to $\phi_i = 172.5^0$ and in this case both faces are visible. Furthermore, from symmetry the magnitude of current on both the

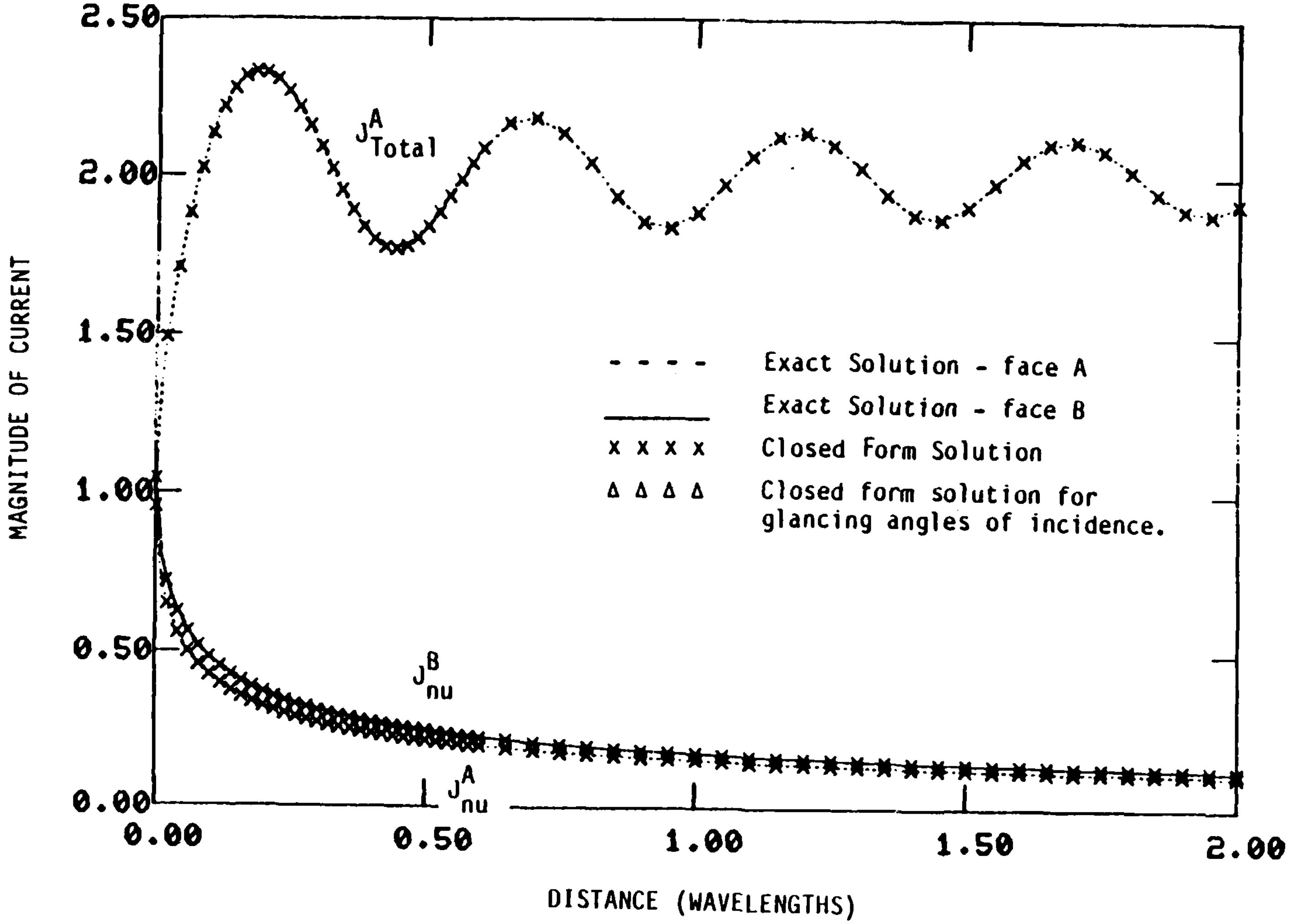

Figure 2. Magnitude of Currents on a Wedge. $\alpha = 15^0$, $\phi_i = 0^0$

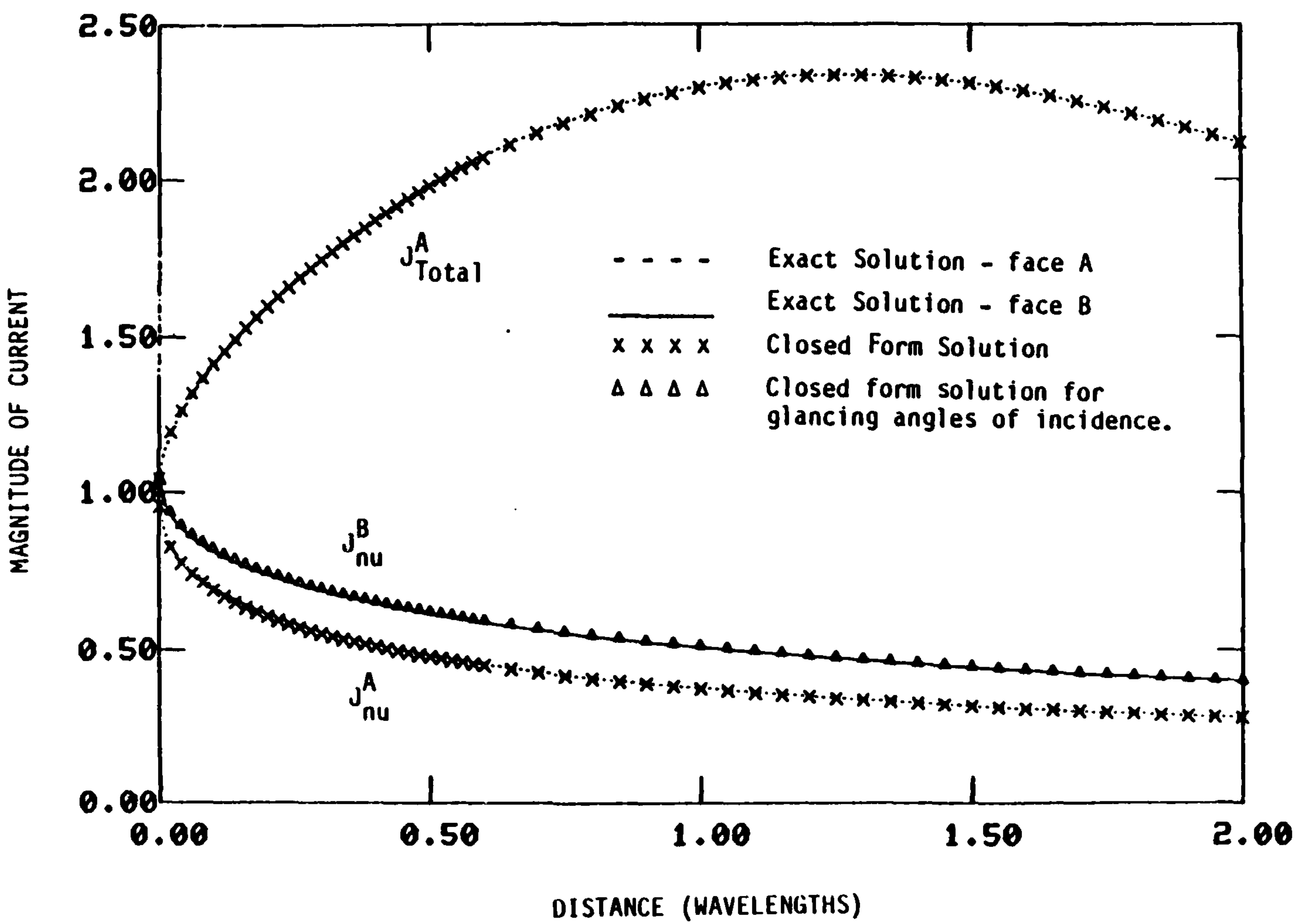

Figure 3. Magnitude of Currents on a Wedge. $\alpha = 15^0$, $\phi_i = 135^0$

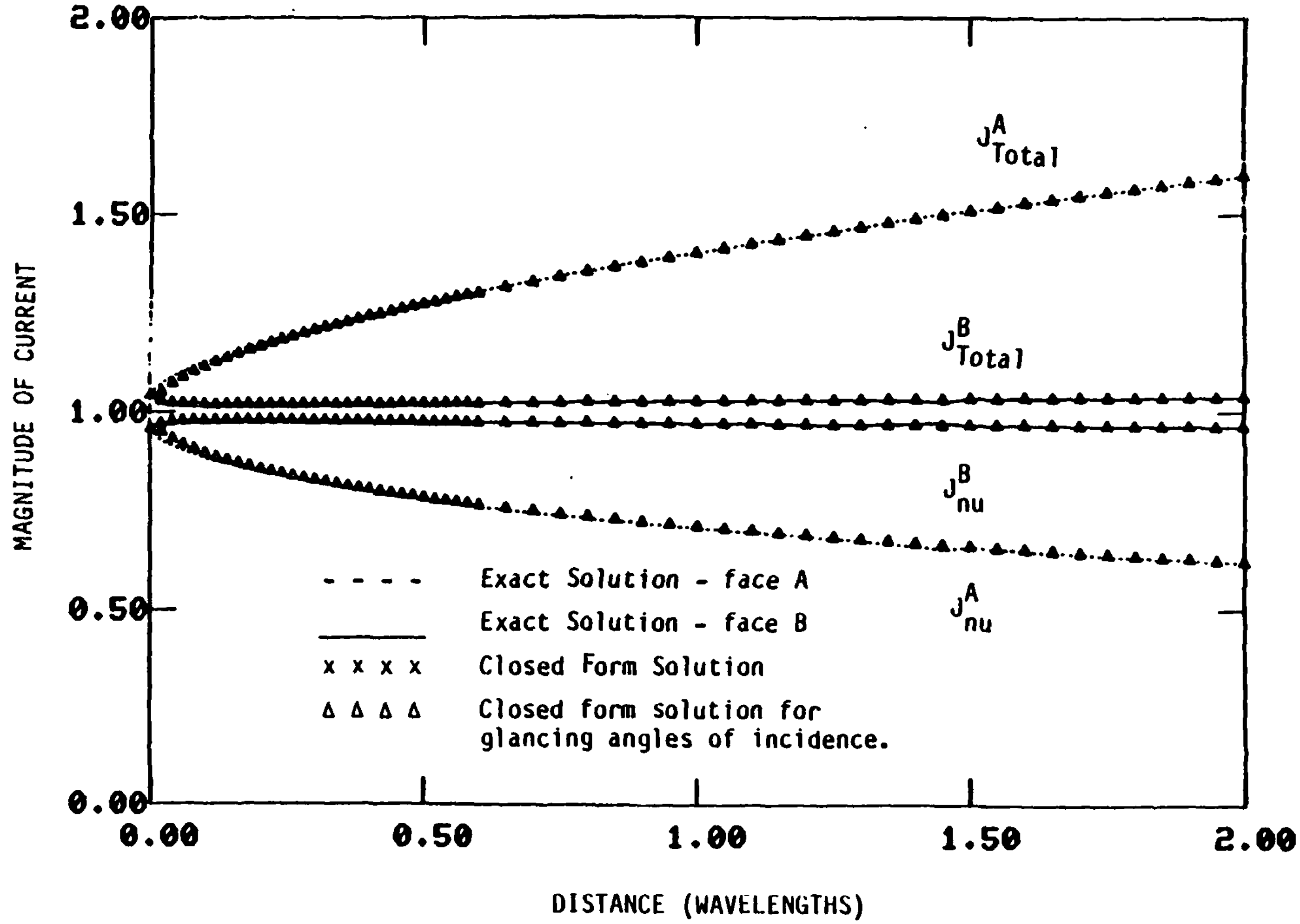

Figure 4. Magnitude of Currents on a Wedge. $\alpha = 15^{0}$, $\psi_i = 166^{0}$

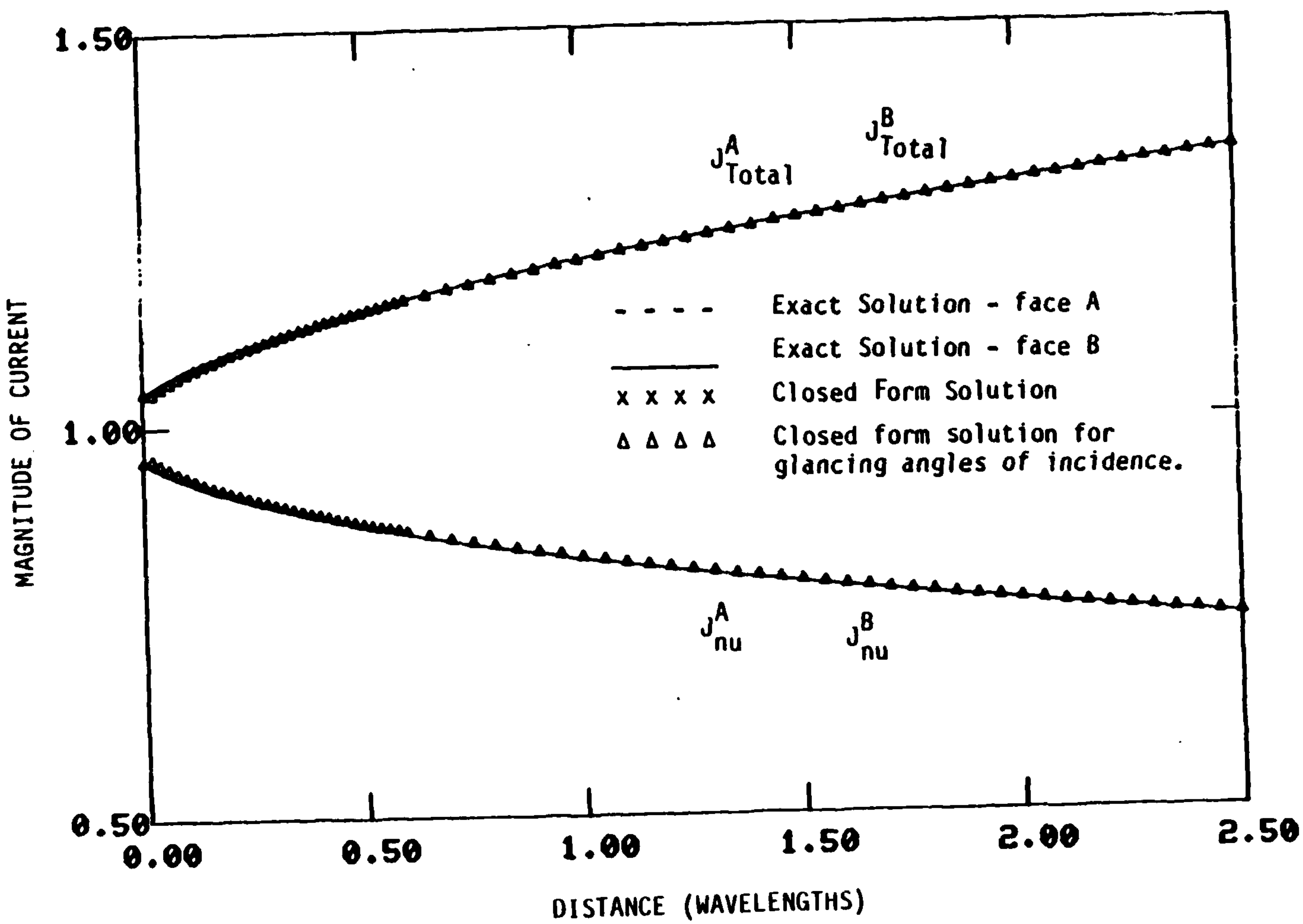

Figure 5. Magnitude of Currents on a Wedge. $\alpha = 15^0$, $\phi_i = 172.5^0$

faces is the same. In Figure 6, we present the phases of non-uniform current when $\phi_i = 166^0$. For the sake of conserving space, we present only one phase plot but note that the agreement shown here is typical. In all of these cases, agreement between our expressions and the exact results is excellent.

Illustration 2: $\alpha = 90^0$

We consider four angles of incidence corresponding to $\phi_i = 0^0$, 22.5^0, 90.01^0 and 135^0. Note that Schretter and Bolle [17] have presented curves corresponding to $\phi_i = 0^0$ and 22.5^0.

Figure 7 and 8 show the currents corresponding to the case $\phi_i = 0^0$ and 22.5^0. Only Face A is illuminated for these cases and currents on neither face is affected by GO boundaries. Figure 9 corresponds to the case of $\phi_i = 90.01^0$. In this case, the reflection boundary is very close to Face B. Figure 10 corresponds to the case of $\phi_i = 135^0$. In view of symmetry, currents on both faces are identical. In Figure 11, we show the phases of total currents on both the faces and note that the degree of agreement shown is typical. It may be pointed out that the agreement between our results and the exact results remains good but not as good as the agreement was for the case of $\phi_i = 15^0$.

Illustration 3: $\alpha = 120^0$

This case has been discussed by Schretter and Bolle [17] and we consider the same angles of incidence, viz., $\phi_i = 15^0$, 90^0 and 120^0 . In addition, we also consider the case for $\phi_i = 55^0$, to illustrate the behavior of the transition region currents.

Figures 12 and 13 correspond to the cases $\phi_i = 15^0$ and 55^0 and in these cases only Face A is illuminated. In the latter case, a shadow boundary is

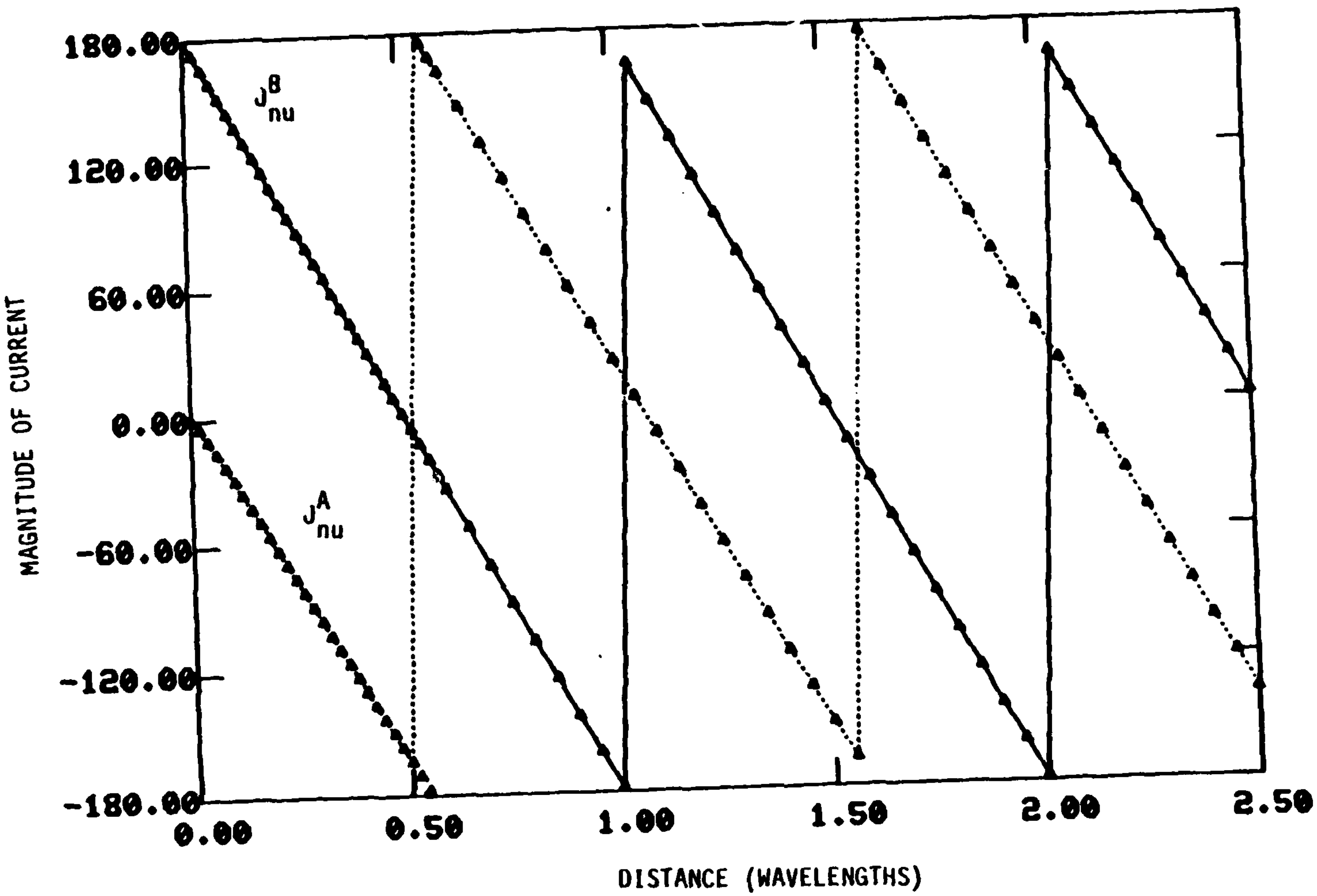

Figure 6. Phase of Currents on a Wedge. $\alpha = 15^0$, $\phi_i = 166^0$

- - - - Exact Solution - face A

―――― Exact Solution - face B

x x x x Closed Form Solution

Δ Δ Δ Δ Closed form solution for glancing angles of incidence.

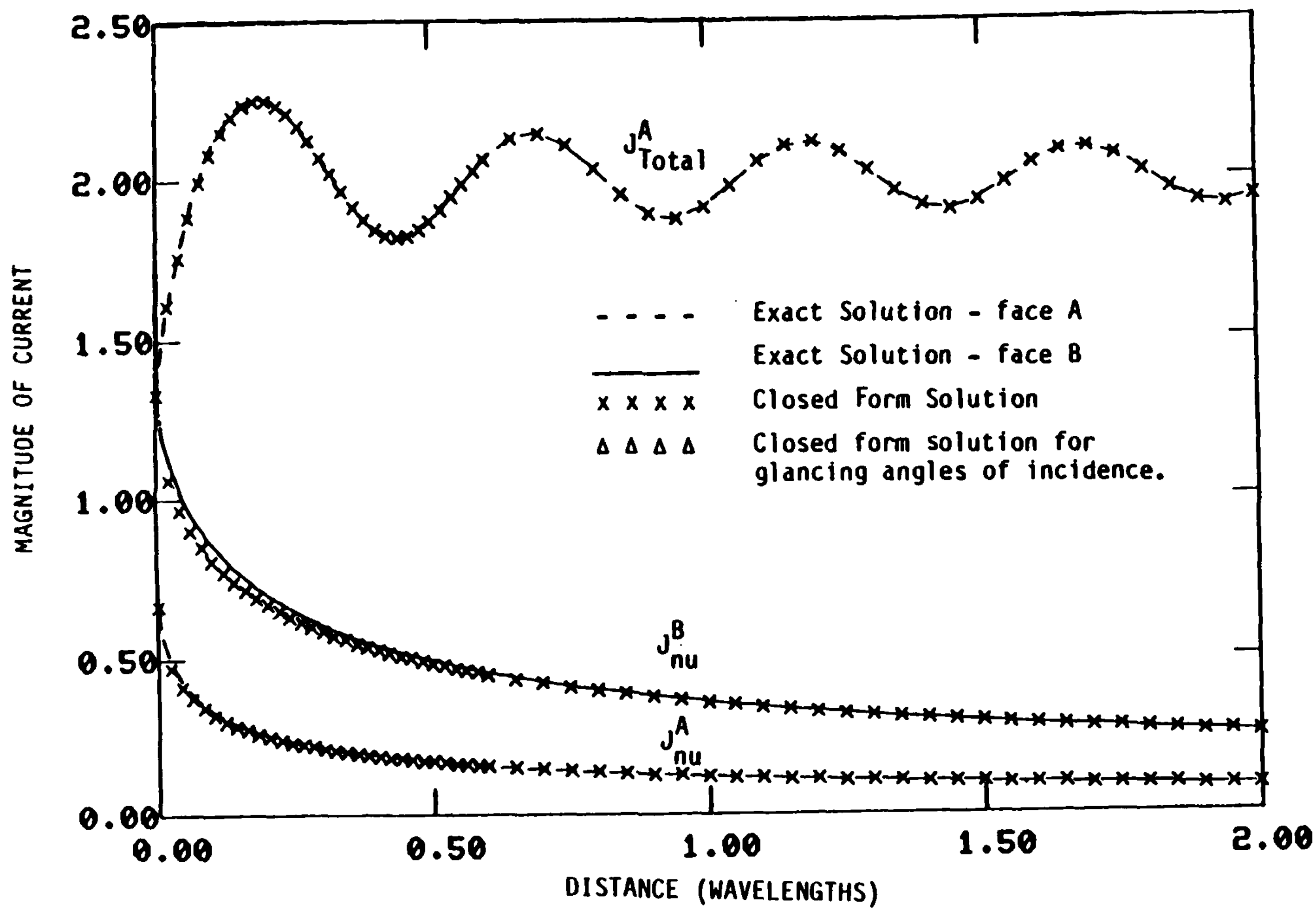

Figure 7. Magnitude of Currents on a Wedge. $\alpha = 90^0$, $\phi_i = 0^0$

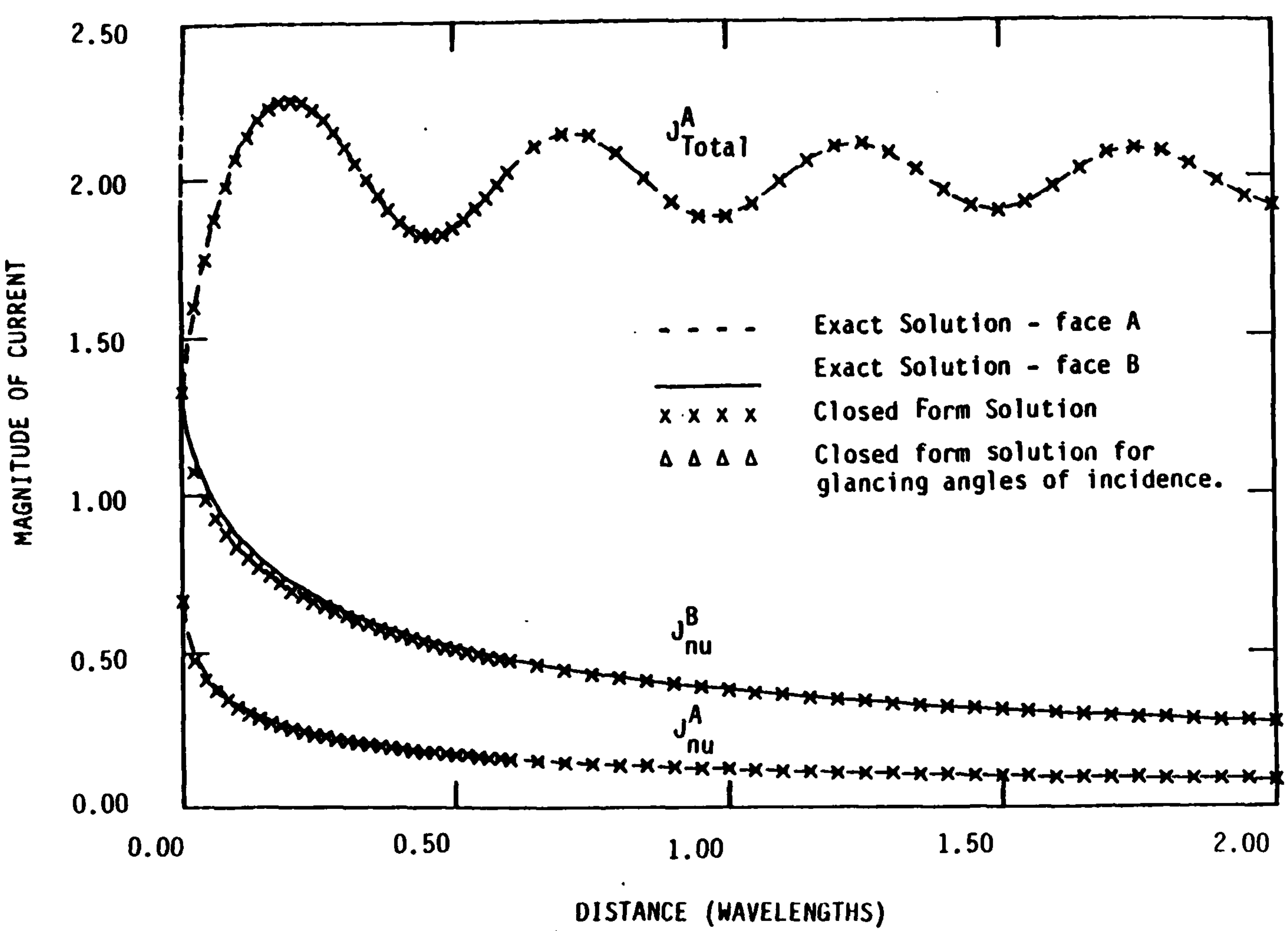

Figure 8. Magnitude of Currents on a Wedge. $\alpha = 90^0$, $\phi_i = 22.5^0$

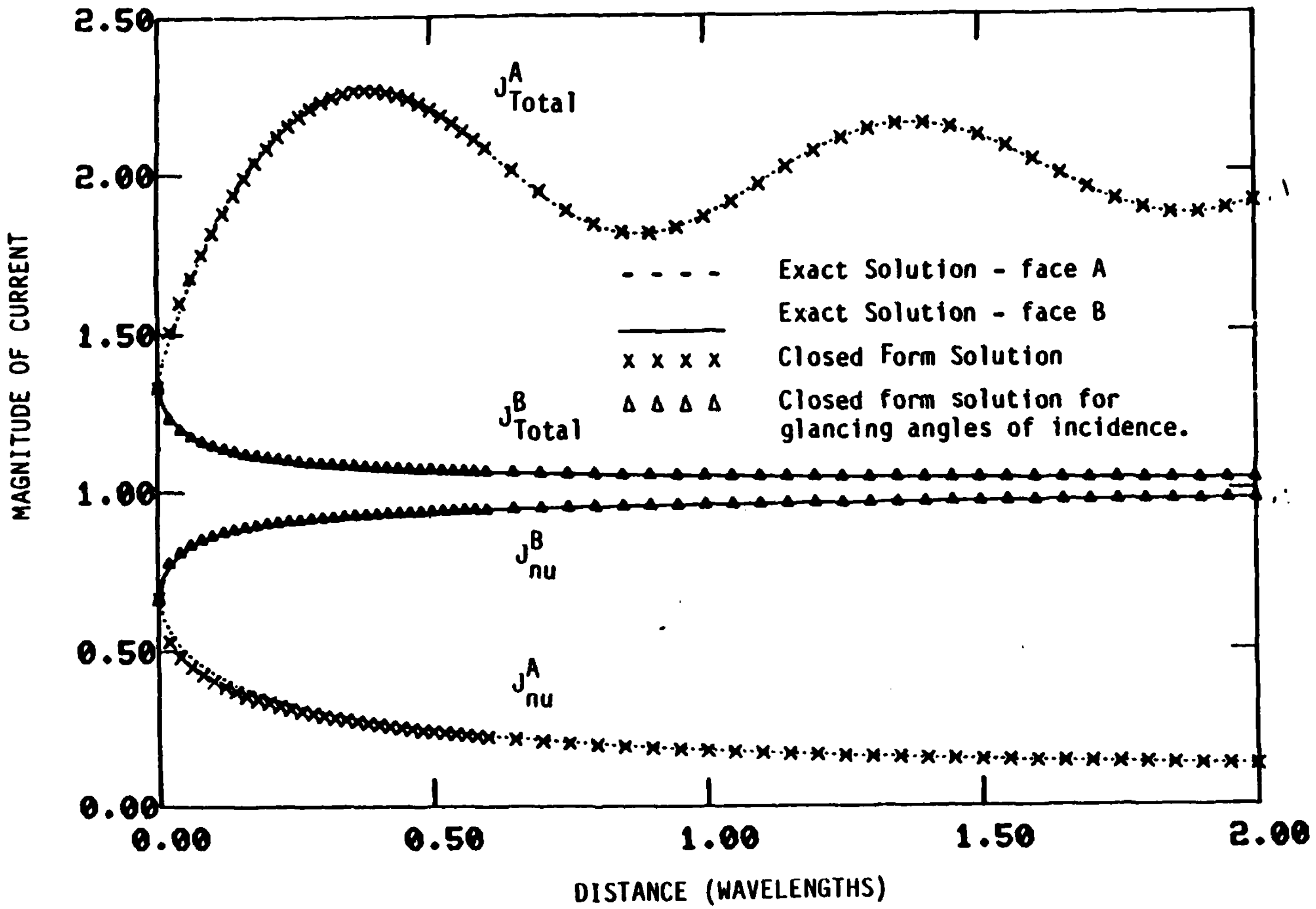

Figure 9. Magnitude of Currents on a Wedge. $\alpha = 90^0$, $\phi_i = 90.01^0$

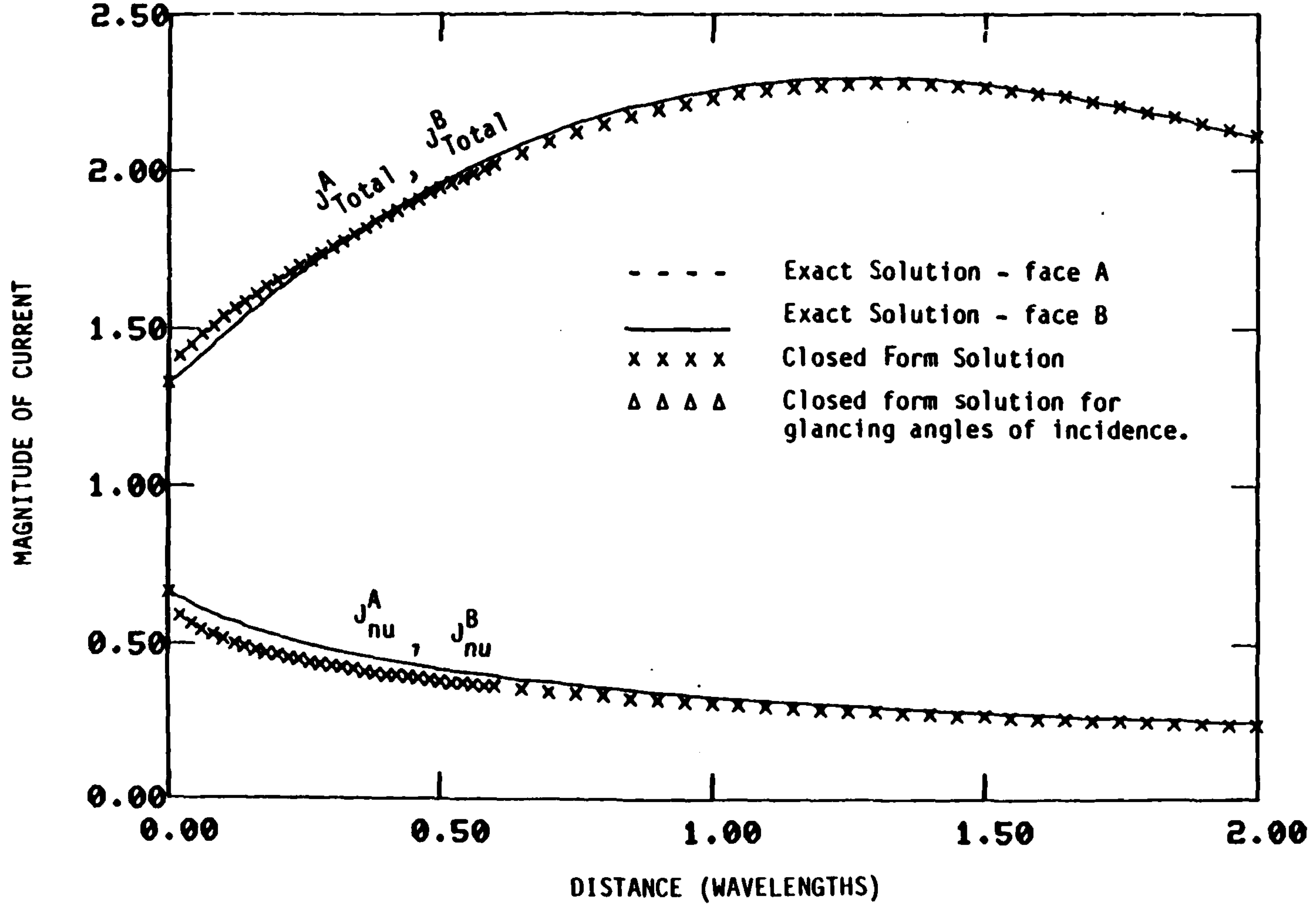

Figure 10. Magnitude of Currents on a Wedge. $\alpha = 90^0$, $\phi_i = 135^0$

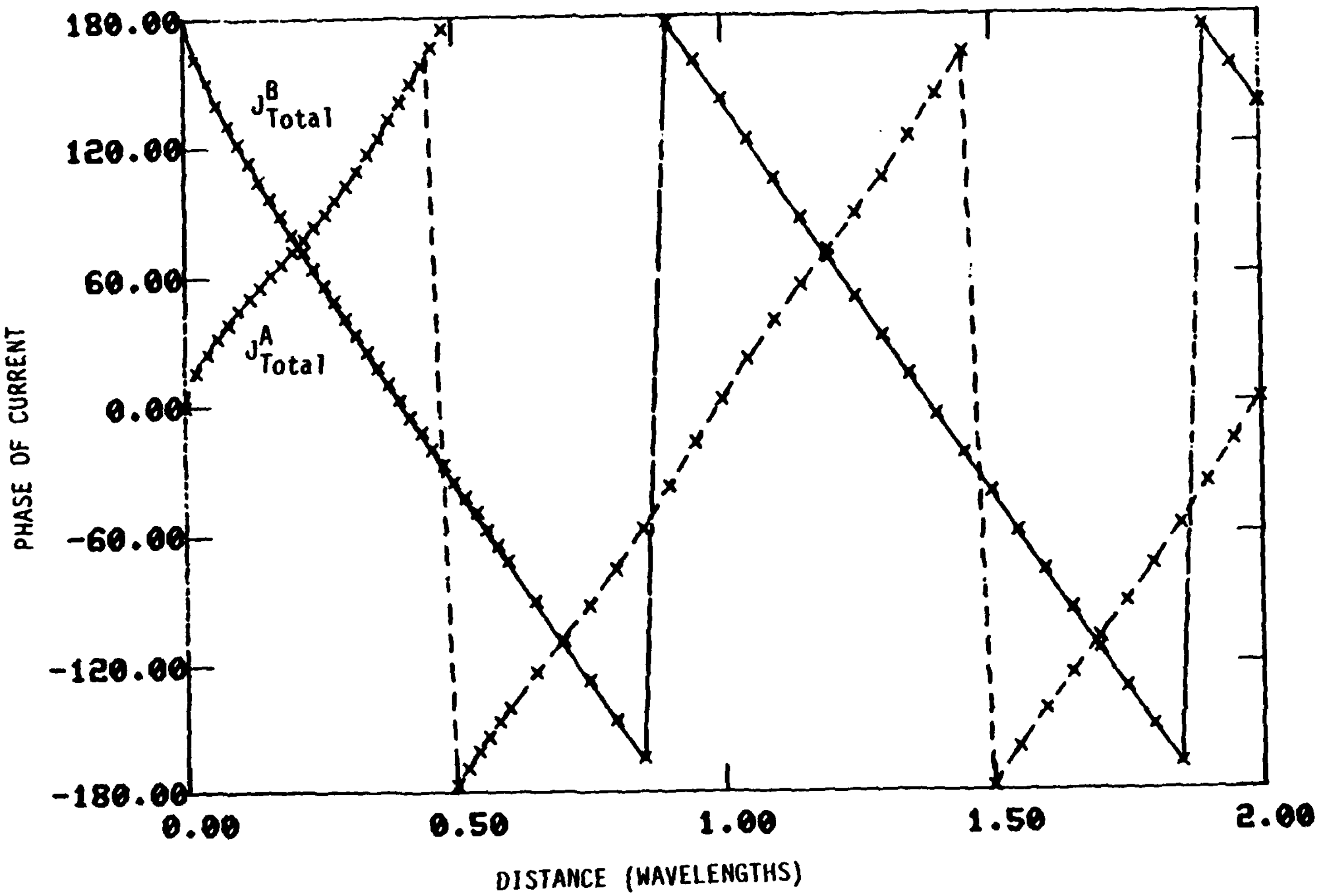

Figure 11. Phase of Currents on a Wedge. $\alpha = 90^0$, $\phi_i = 135^0$

- - - - Exact Solution - face A

——— Exact Solution - face B

x x x x Closed Form Solution

Δ Δ Δ Δ Closed form solution for glancing angles of incidence.

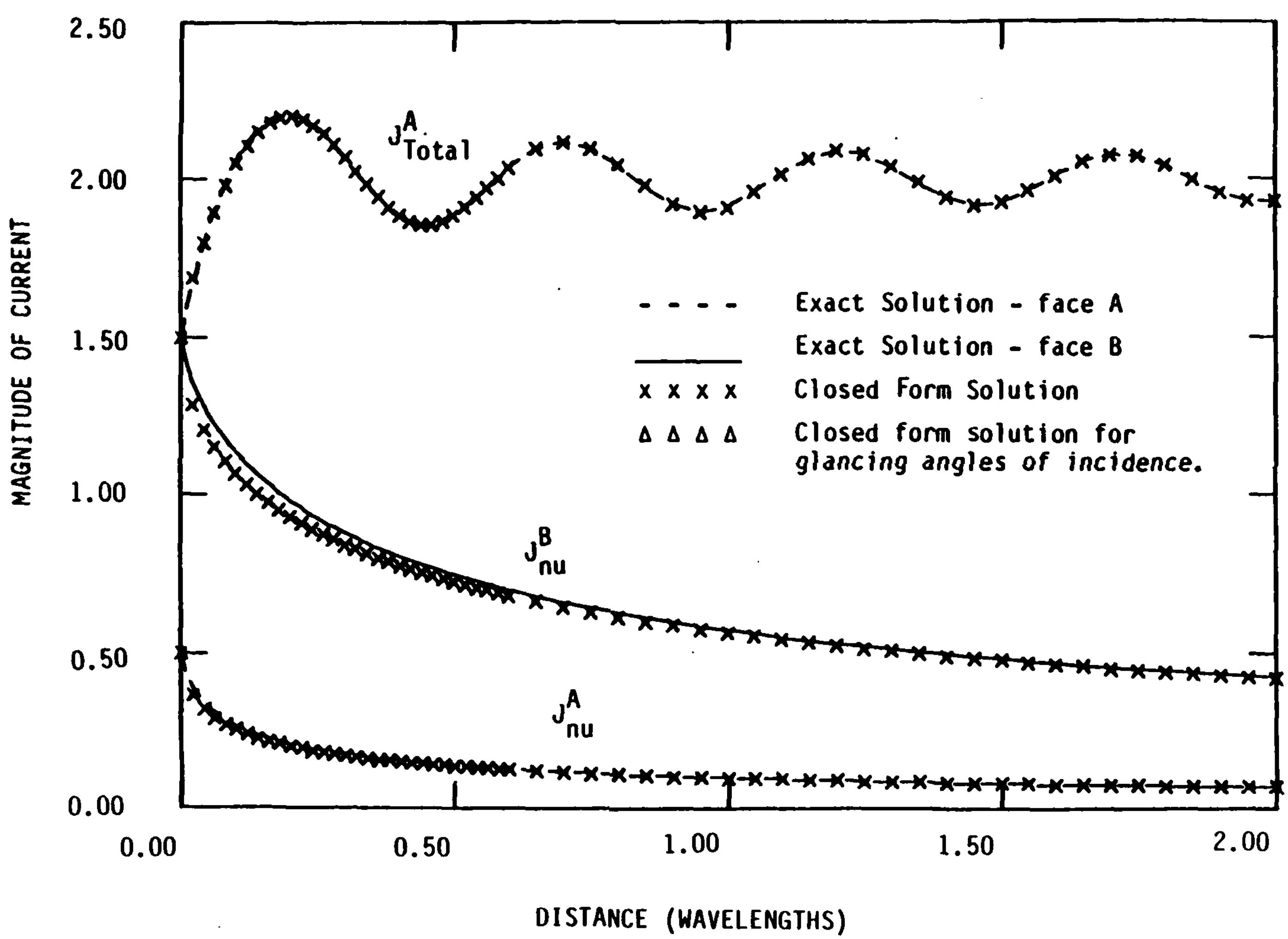

Figure 12. Magnitude of Currents on a Wedge. $\alpha = 120^0$, $\phi_i = 15^0$

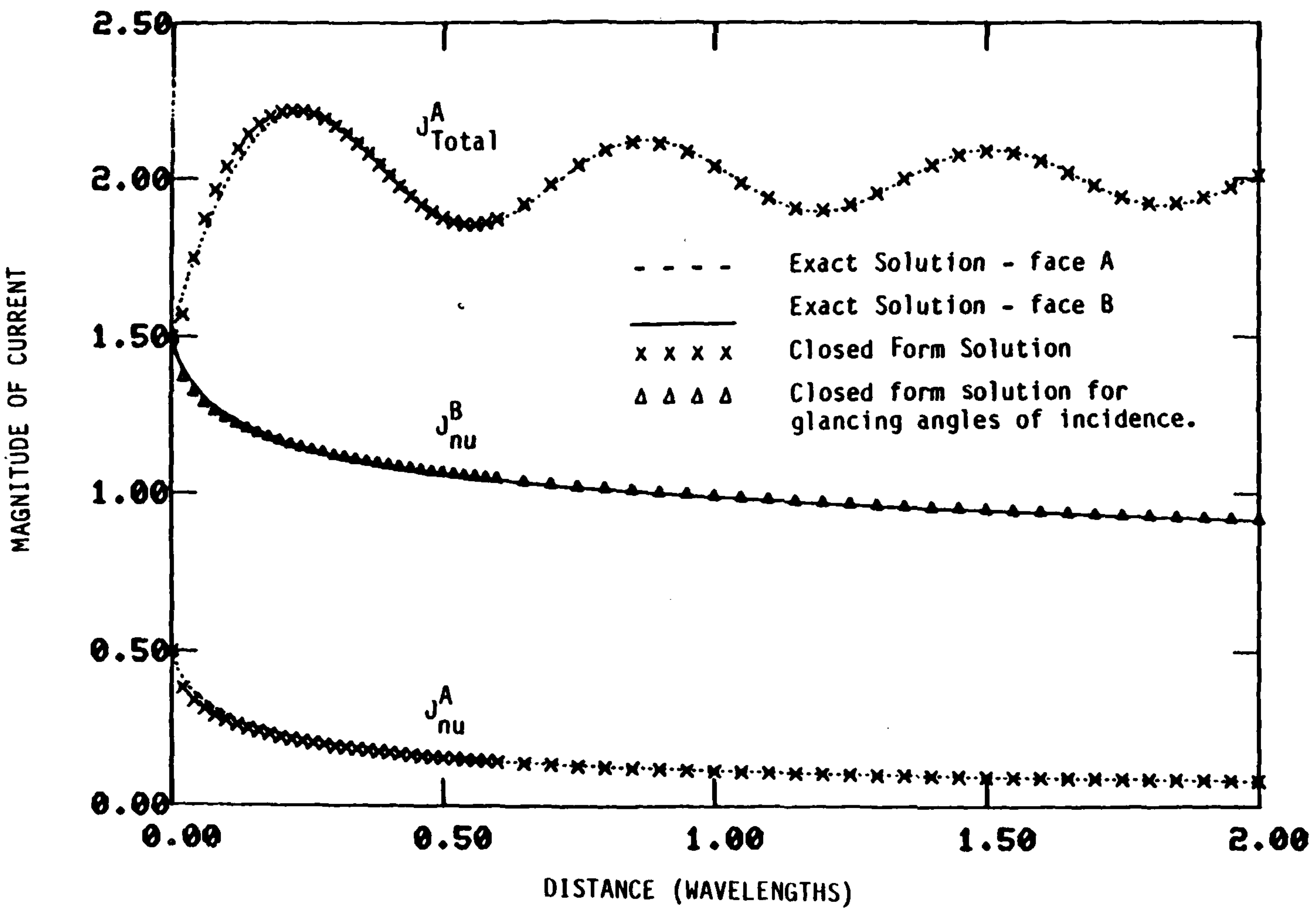

Figure 13. Magnitude of Currents on a Wedge. $\alpha = 120^0$, $\phi_i = 55^0$

close to Face B. Figures 14 and 15 correspond to $\phi_i = 90^0$ and 120^0. These angles of incidence are such that both faces are illuminated and further, in the latter case, the currents on both faces are identical.

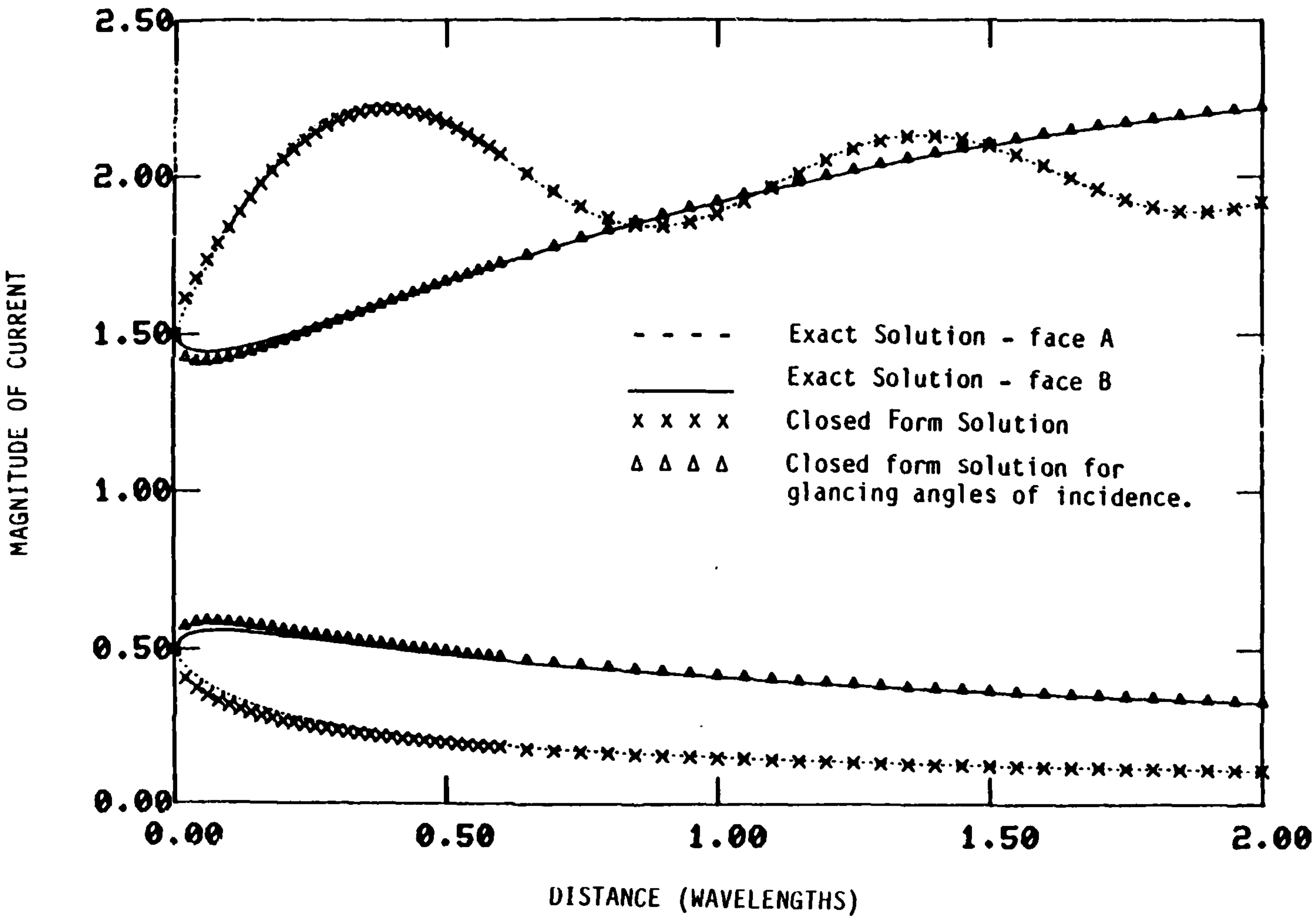

Figure 14. Magnitude of Currents on a Wedge. $\alpha = 120^0$, $\phi_i = 90^0$

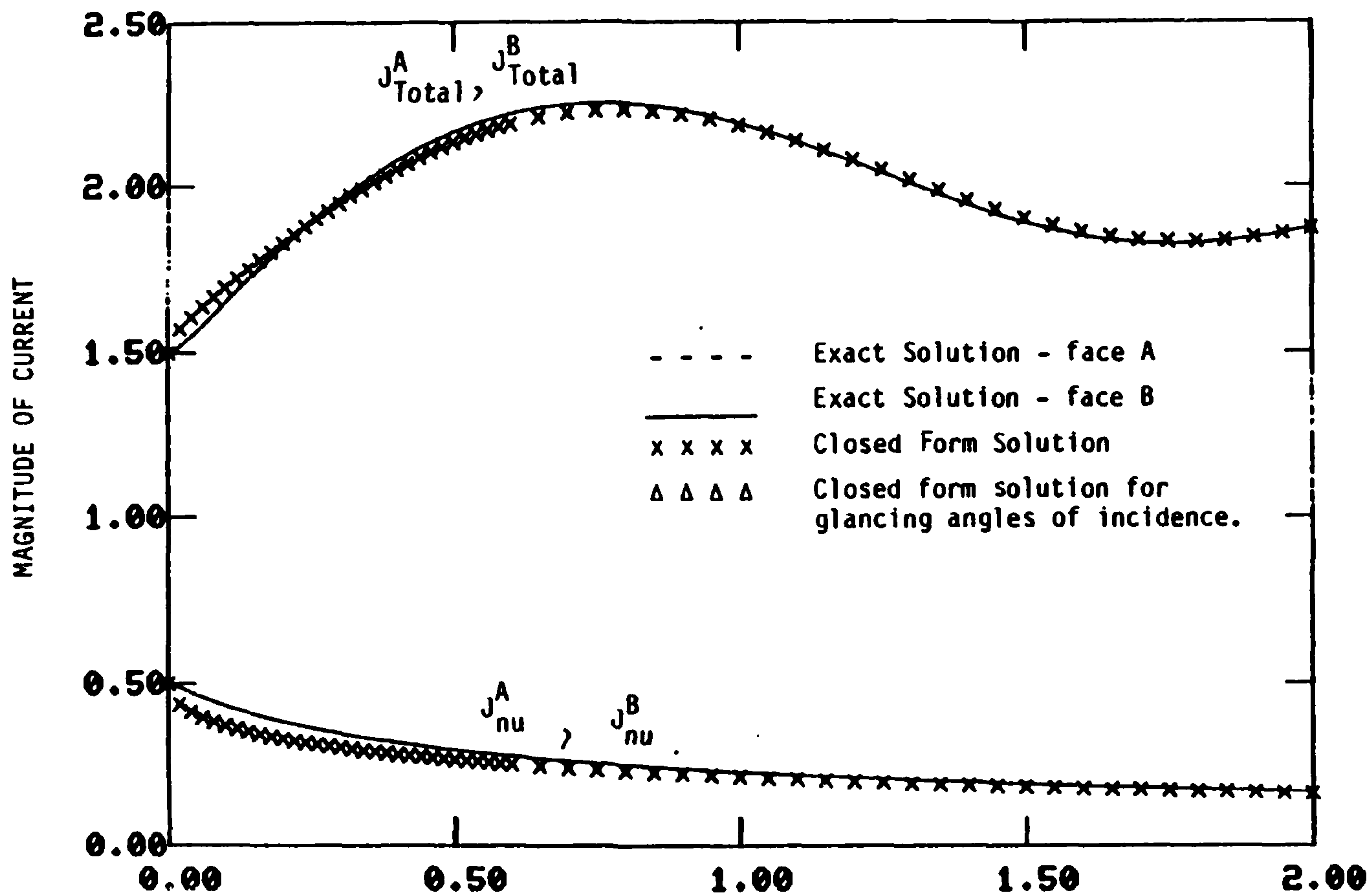

Figure 15. Magnitude of Currents on a Wedge. $\alpha = 120^0$, $\phi_i = 120^0$

4. DISCUSSION AND CONCLUDING REMARKS

In this paper we present closed form expressions for non-uniform currents on a perfectly conducting wedge illuminated by a TE-plane wave. These expressions are valid for all wedge angles and any given angle of incidence, but are not uniform. The expressions derived in this paper involve the simple and well-known modified Fresnel functions. These expressions are such that far from the edge, they agree with the asymptotic theory of diffraction and at the edge, they agree with the eigenfunction solution. Furthermore, as demonstrated by numerical illustrations, for intermediate distances from the edge, our expressions agree quite well with the exact solution. Exact closed form solutions are available in the literature for the cases of a half plane and an infinite plane. Our solution reduces to these exact expressions for those special cases.

It is often suggested that the non-uniform currents are significant only in the immediate vicinity of the edge [17]. This is true only when the wedge faces are *not* in the vicinity of a GO boundary. As demonstrated by the illustrations in the preceding section (Figures 4, 9 and 13, for instance), transition region currents remain significant far from the edge.

There are several advantages in having closed form expressions for non-uniform currents. As pointed out by Schretter and Bolle [17], it becomes possible to take the Fourier transform of the current and, consequently, simplifies problems involving pulse scattering from edged bodies. Recently, these expressions have been incorporated into an iterative scheme for solving the magnetic field integral equation with a considerable success [22]. It is also possible to use these expressions to obtain the fields in a caustic region as an integral of surface currents, similar to the technique of MEC.

It would be interesting to obtain expressions for currents that are uniformly valid. Also, the case of excitation of a wedge by a TM plane wave needs to be considered. These problems are presently under investigation and the results would be communicated in due course.

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